$$y^n(t) = C_1 e^{-t} + C_2 t e^{-t}$$

Now

$$y(0) = 0$$

$$\therefore \quad 0 = C_1 e^0 + C_2(0) c^0$$

$$\boxed{C_1 = 0}$$

then
$$\frac{dy^n(t)}{dt} = C_2\left(-t e^{-t} + e^{-t}\right)$$

$$\frac{dy^n(0)}{dt} = 7 = C_2(0 + 1)$$

$$\therefore C_2 = 7$$

$$\boxed{y^n(t) = 7 t e^{-t}}$$

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3. Find complete solution i/p $x[n] = \left(\frac{1}{2}\right)^n u(n)$, $y[-1] = 8$

$$y[n] - \frac{1}{4} y[n-1] = x[n] \quad \text{——— (1)}$$

Sol^n Homogeneous eq. is $y[n] - \frac{1}{4} y[n-1] = 0$ ——— (2)

$$\therefore \quad y^h[n] = C_1 \gamma_1^n$$

Characteristic eq^n

$$a_0 \gamma^n + a_1 \gamma^{n-1} = 0$$

$$\gamma^n - \frac{1}{4} \gamma^{n-1} = 0$$

$$\gamma = 1/4$$

$$\therefore \quad y^h[n] = C_1\left(\frac{1}{4}\right)^n \quad \text{——— (3)}$$

Particular Solution :-

Let $y^p[n] = \left(\frac{1}{2}\right)^n c$ — (4)

substituting in eq^n (1)

$$c\left(\frac{1}{2}\right)^n - c\,\frac{1}{4}\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n$$

$$c = 2$$

$$\therefore \quad y^p[n] = 2\left(\frac{1}{2}\right)^n$$

$\therefore$ Complete solution is

$$y[n] = C_1\left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n$$

substituting initial condition

$$y[-1] = 8$$

$$y[-1] = C_1\left(\frac{1}{4}\right)^{-1} + 2\left(\frac{1}{2}\right)^{-1}$$

$$8 = 4C_1 + 4$$

$$C_1 = 1$$

$$\therefore \quad y[n] = \left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n$$